Towards experimentally testing the paradox of black hole information loss

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Information about the collapsed matter in a black hole will be lost if Hawking radiations are truly thermal. Recent studies discover that information can be transmitted from a black hole by Hawking radiations, due to their spectrum deviating from exact thermality when back reaction is considered. In this paper, we focus on the spectroscopic features of Hawking radiation from a Schwarzschild black hole, contrasting the differences between the nonthermal and thermal spectra. Of great interest, we find that the energy covariances of Hawking radiations for the thermal spectrum are exactly zero, while the energy covariances are non-trivial for the nonthermal spectrum. Consequently, the nonthermal spectrum can be distinguished from the thermal one by counting the energy covariances of successive emissions, which provides an avenue towards experimentally testing the long-standing "information loss paradox".

I. INTRODUCTION

Although not yet confirmed experimentally, it is widely believed that a black hole evaporates almost like an ideal black body. This evaporation gives rise to the famous Hawking radiation [1, 2], where a thermal spectrum [3–5] leads to a crisis of quantum physics [6, 7], the so-called "information loss paradox". In this scenario, information is lost as a black hole evaporates, inconsistent with the unitarity of quantum mechanics, which thus presents a serious obstacle for developing theories of quantum gravity. Many solutions aimed at reconciling a thermal Hawking radiation spectrum with unitarity were discussed [8–13], none is capable of successfully ending the dispute. As was pointed out by Wilczek and his collaborators [14, 15], the thermal spectrum is due to the neglect of back reaction from emitted radiation in Hawking's original treatment. An improved derivation even at a semiclassical level gives a spectrum slightly deviates from the thermal one when energy conservation is enforced during the emission process [14–16].

With the nonthermal spectrum of Parikh and Wilczek, we recently show that Hawking radiations can carry off all information about the collapsed matter in a black hole [17, 18]. After revealing the existence of information-carrying correlation among Hawking radiations when the spectrum is nonthermal, we show that entropy is conserved for Hawking radiation based on standard probability theory and statistics. This prompts us to claim that the information previously considered lost is hidden inside Hawking radiation, or encoded into correlations. Our study thus establishes the basis for a signifi-

cant way towards resolving the long-standing information loss paradox [19].

After discovering that the nonthermal spectrum of Parikh and Wilczek allows for the emissions to carry off all information of a black hole [17–19], a natural question arises as to whether Hawking radiation is indeed nonthermal or not? Although the derivation of the nonthermal spectrum is based on solid theoretical background, it remains to be confirmed experimentally or in observations. In this paper, we explore experimental signatures associated with a nonthermal spectrum of Hawking radiation. Of great interest, we establish smoking gun like signatures capable of distinguishing the slightly deviated nonthermal spectrum from the thermal spectrum, based on an extensive analysis on the differences between the two spectra. We thus proclaim that the information-carrying correlations among Hawking radiations can be confirmed by observing the energy covariances of successive emissions. Finally, we analyze the possibility of testing experimentally the paradox by counting Hawking radiations.

II. UNCOVERING THE LOST INFORMATION

We start by briefly reviewing our resolution to the paradox [17, 18]. The nonthermal spectrum of Parikh and Wilczek [15] for a Schwarzschild black hole with mass $\mathcal{M}$ is given by

$$\Gamma_{\mathrm{NT}} = \exp\left[-8\pi\mathcal{E}\left(\mathcal{M} - \frac{\mathcal{E}}{2}\right)\right]. \tag{1}$$

From our earlier analysis [17], we use $S_{\mathrm{NT}}\left(\mathcal{E}_i|\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_{i-1}\right)$ to denote the entropy of a Hawking emission, at an energy $\mathcal{E}_i$, conditional on the earlier emissions labeled by $\mathcal{E}_1, \mathcal{E}_2, \cdots,$ and $\mathcal{E}_{i-1}$. As

shown before, it is easy to compute that

$$S_{\mathrm{NT}}\left(\mathcal{E}_i|\mathcal{E}_1,\mathcal{E}_2,\cdots,\mathcal{E}_{i-1}\right) = 8\pi\mathcal{E}_i\left(\mathcal{M}-\sum_{j=1}^{i-1}\mathcal{E}_j-\mathcal{E}_i/2\right). \tag{2}$$

Thus the total entropy for a given sequence of emissions $(\mathcal{E}_1,\mathcal{E}_2,\cdots,\mathcal{E}_n)$ that exhaust the initial black hole ($\mathcal{M}=\sum_{i=1}^{n}\mathcal{E}_i$) is given by

$$\begin{aligned}\mathcal{S}_{\mathrm{NT}}\left(\mathcal{E}_1,\mathcal{E}_2,\cdots,\mathcal{E}_n\right) &= \sum_{i=1}^{n}S_{\mathrm{NT}}\left(\mathcal{E}_i|\mathcal{E}_1,\mathcal{E}_2,\cdots,\mathcal{E}_{i-1}\right)\\ &= 4\pi\mathcal{M}^2\\ &= S_{\mathrm{BH}}, \end{aligned} \tag{3}$$

where S_{BH} is the celebrated Bekenstein-Hawking entropy for a black hole [20]. This equality states the entropy of all emitted Hawking radiation is equal to the entropy of the initial black hole, which implies no information is lost in the process of Hawking radiation [17–19].

III. CORRELATION AMONG HAWKING RADIATIONS

With the nonthermal spectrum, correlations among Hawking radiations are calibrated as follows: first we calculate the correlation between the emissions of energies $\mathcal{E}_1$ and $\mathcal{E}_2$ and the result is $8\pi\mathcal{E}_1\mathcal{E}_2$ [17, 18]; we then calculate the correlation between the emissions with energies $\mathcal{E}_1+\mathcal{E}_2$ and $\mathcal{E}_3$ and the result becomes $8\pi\left(\mathcal{E}_1+\mathcal{E}_2\right)\mathcal{E}_3$; the total correlation among the three emissions $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$, thus becomes $8\pi\mathcal{E}_1\mathcal{E}_2+8\pi\left(\mathcal{E}_1+\mathcal{E}_2\right)\mathcal{E}_3$, which does not depend on the orders of the individual emissions. For the nonthermal spectrum, the total correlation in a queue of Hawking radiations is summed up to

$$\mathcal{C}_{\mathrm{NT}} = \sum_{l\geq2}8\pi\left(\mathcal{E}_1+\mathcal{E}_2+\cdots\mathcal{E}_{l-1}\right)\mathcal{E}_l = \sum_{i>j}8\pi\mathcal{E}_i\mathcal{E}_j, \tag{4}$$

where indices i,j take all possible values labeling emissions. Clearly, the analogous correlation vanishes for a thermal spectrum, or $\mathcal{C}_{\mathrm{T}}=0$ [6]. This will be discussed later as it plays a crucial role in experimentally resolving the information loss paradox.

IV. TWO SPECTRA FEATURES COMPARED

Given a queue of emissions, we can compute the average energies and the covariances of the emitted radiations for the two spectra: thermal or nonthermal. These are studied below after we introduce suitable units for discussing the physics of Hawking radiations and their associated properties. Normal treatment of Hawking radiation assumes a static thermal black body emission. Because the emission corresponds to a slow evaporation process for most black holes, we can include the time
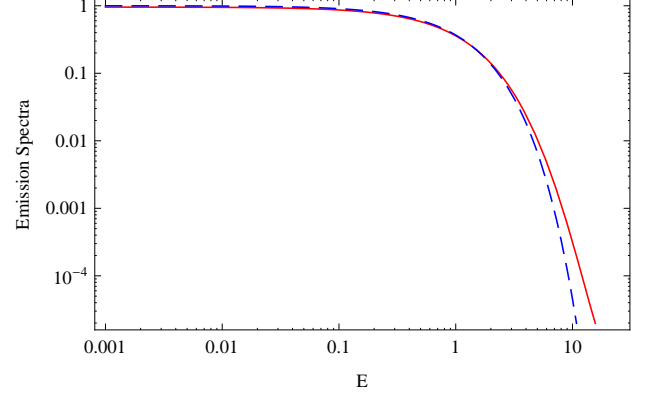

FIG. 1: (Color online) The two spectra compared for a black hole at the Planck mass scale. The red solid (blue dashed) line refers to the nonthermal (thermal) spectrum.

dependence adiabatically. Using the Stefan-Boltzmann's law for black body thermal emission and assuming no other mass accretion or evaporation mechanisms exist, we can relate the reduction of the mass for a black hole to the energies of the emissions according to

$$\frac{d\mathcal{M}}{dt} = -4\pi R^2\sigma T^4, \tag{5}$$

where $R=2\mathcal{M}$ is the Schwarzschild radius, the temperature of Hawking radiation is given by $T=1/8\pi\mathcal{M}$, inversely proportional to its mass, and the Stefan-Boltzmann constant $\sigma=2\pi^5k_b^4/(15c^2h^3)$. This gives the steady state time dependent mass

$$\mathcal{M}\left(t\right) = \mathcal{M}_0 - \left(\frac{3\sigma}{256\pi^3}\right)^{\frac{1}{3}}\left(t-t_0\right)^{\frac{1}{3}}, \tag{6}$$

of a black hole if the total mass change is due to Hawking radiation alone. In the above, $\mathcal{M}_0$ denotes the initial mass of a black hole at time t_0. In subsequent emissions, the mass thus becomes a time unit according to Eq. (6). Therefore, some of the figures illustrated below are plotted against mass instead of time.

To prepare for numerically comparison of the observables associated with the two spectra, we normalize both according to $\int_0^{\mathcal{M}}\Gamma(\mathcal{E})d\mathcal{E}=1$ and obtain respectively the normalized thermal spectrum

$$\Gamma_{\mathrm{T}}\left(\mathcal{E}\right) = 8\pi\mathcal{M}^2\exp\left(-8\pi\mathcal{M}\mathcal{E}\right)/[1-\exp\left(-8\pi\mathcal{M}^2\right)], \tag{7}$$

and the nonthermal spectrum

$$\Gamma_{\mathrm{NT}}\left(\mathcal{E}\right) = 2\sqrt{\pi}\mathcal{M}\exp\left(-8\pi\mathcal{E}\left(M-\mathcal{E}/2\right)\right)/\mathrm{F}(2\sqrt{\pi}\mathcal{M}), \tag{8}$$

where $\mathrm{F}(x)$ is the Dawson function [21]. We can now discuss the relevant features of the two spectra in quantitative terms. We will illustrate their differences graphically. As a result of quantum mechanics meeting with gravity, the spectrum for Hawking radiation is a function
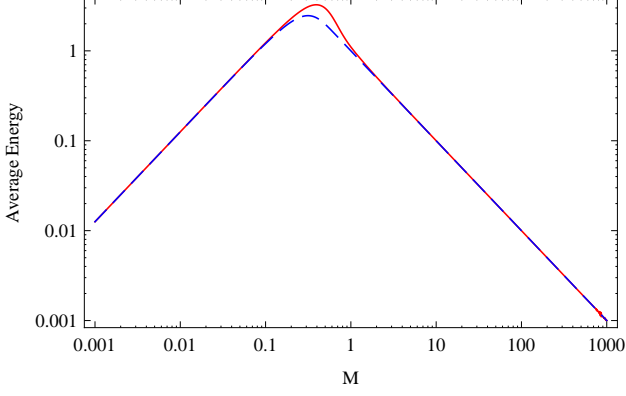
FIG. 2: (Color online) The average emission energies compared, as in Fig. 1.

FIG. 3: (Color online) The average number of emissions compared for the two spectra as in Fig. 1.

of physical quantities from very large (c) to very small ($\hbar$ and G). Therefore, it is more convenient for numerical purposes to adopt real units instead of the commonly used units of $G = c = \hbar = 1$.

Introducing the dimensionless mass $M = \mathcal{M}/\mathcal{M}_{\mathrm{pl}}$ and energy $E = \mathcal{E}/k_b T_{\mathrm{pl}}$ in terms of Planck mass $\mathcal{M}_{\mathrm{pl}}$ and Planck temperature T_{pl}, we arrive at the respectively normalized spectra

$$\Gamma_{\mathrm{NT}}(E) = \exp\left[-(M - E/16\pi)E\right]/4\sqrt{\pi}\mathrm{F}(2\sqrt{\pi}M), \tag{9}$$

and

$$\Gamma_{\mathrm{T}}(E) = M\exp(-ME)/[1 - \exp(-8\pi M^2)], \tag{10}$$

with $E \in [0, 8\pi M]$. Figure 1 compares the two spectra for a black hole at Planck scale. Their difference is concentrated near $E \sim k_b T_M$, where T_M denotes the equivalent Hawking radiation temperature for a black hole of mass $\mathcal{M}$ measured in units of T_{pl}.

In units of $k_b T_{\mathrm{pl}}$, the average energy of Hawking emissions can be computed at any instant, as for a fixed mass black hole. For the thermal spectrum, we find

$$\langle E(M)\rangle_{\mathrm{T}} = \left(M + \frac{8\pi M^3}{e^{8\pi M^2} - 1 - 8\pi M^2}\right)^{-1}, \tag{11}$$

which approaches $4\pi M$ for $M \ll 1$, and $1/M$ for $M \gg 1$. This is to be compared with the nonthermal spectrum, for which we find

$$\langle E(M)\rangle_{\mathrm{NT}} = 8\pi M - \frac{2\sqrt{\pi}(1 - e^{-4\pi M^2})}{\mathrm{F}(2\sqrt{\pi}M)}. \tag{12}$$

Interestingly both limits are the same as for the thermal spectrum discussed above for $M \ll 1$ and $M \gg 1$. Their noticeable difference exists only near the Planck scale as shown in Fig. 2.

The average number of radiations emitted from a black hole with mass M can be obtained for the thermal spectrum approximately as
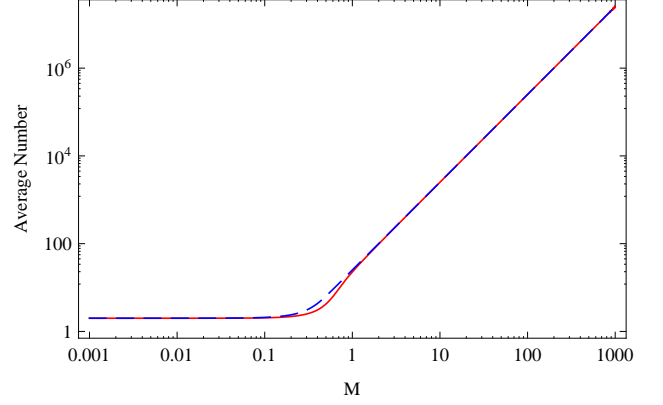
$$N_{\mathrm{T}}(M) = \frac{8\pi M}{\langle E(M)\rangle_{\mathrm{T}}} = \frac{8\pi\left(e^{8\pi M^2} - 1 - 8\pi M^2\right)}{e^{8\pi M^2} - 1}, \tag{13}$$

while for the nonthermal spectrum we find

$$N_{\mathrm{NT}}(M) = \frac{4\pi M\mathrm{F}(2\sqrt{\pi}M)}{4\pi M\mathrm{F}(2\sqrt{\pi}M) - \sqrt{\pi}(1 - e^{-4\pi M^2})}. \tag{14}$$

Again both Eqs. (13) (14) give the same limits: 2 for $M \ll 1$, and $8\pi M^2$ for $M \gg 1$. As shown in Fig. 3, the average number of emissions increase rapidly with the black hole mass. Below the Planck scale, however, the average number of emissions remains nearly a constant.

We now compute the standard deviations of the emission energies, and we find for the thermal spectrum

$$\begin{aligned}
\delta E_{\mathrm{T}}^2(M) &= \langle E^2(M)\rangle_{\mathrm{T}} - \langle E(M)\rangle_{\mathrm{T}}^2 \\
&= \frac{(\cosh 8\pi M^2 - 1 - 32\pi^2 M^4)\mathrm{csch}^2 4\pi M^2}{2M^2},
\end{aligned} \tag{15}$$

whose large and small M limits are given respectively by $16\pi^2 M^2/3$ for $M \ll 1$, and $1/M^2$ for $M \gg 1$. For the nonthermal spectrum, we find

$$\begin{aligned}
\delta E_{\mathrm{NT}}^2(M) = \frac{4\pi}{\mathrm{F}^2(2\sqrt{\pi}M)}\Big[&4\sqrt{\pi}M\mathrm{F}(2\sqrt{\pi}M) \\
&- (1 - e^{-4\pi M^2})^2\Big] - 8\pi,
\end{aligned} \tag{16}$$

again with the same large and small M limits as for the thermal spectrum. Figure 4 shows clearly these features and illustrates the dependence of the variances on the average energy.

Based on our extensive analysis, we find no drastic differences between the two spectra except for tiny black holes with masses near the Planck scale. Naively, one
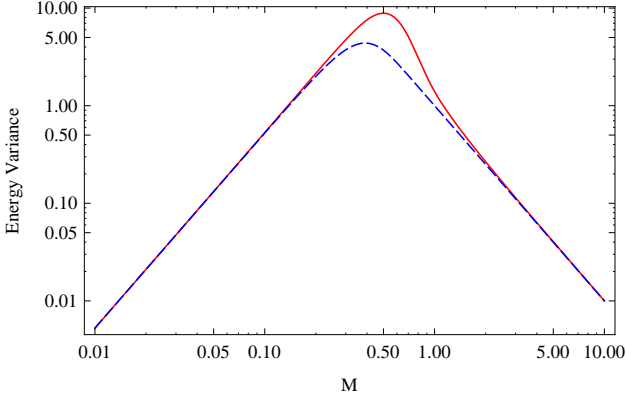
FIG. 4: (Color online) The radiation energy variances compared for the two spectra as in Fig. 1.

would conclude that it is therefore essentially impossible to experimentally distinguish the nonthermal spectrum from the thermal one, let alone to query about whether information can be carried out from a black hole by correlations hidden in the emissions. Nevertheless, we demonstrate below that information stored in the correlations of Hawking radiations from the nonthermal spectrum can indeed be observed through a counting of the emission energy covariances.

V. ENERGY COVARIANCES

For the thermal spectrum, individual emissions are uncorrelated [6], thus one expects a vanishing covariance. This is indeed what we find

$$\delta E_{\mathrm{T}}^{2(\mathrm{cov})} = \langle E_i(M)E_{j\neq i}(M)\rangle_{\mathrm{T}} - \langle E_i(M)\rangle_{\mathrm{T}}\langle E_{j\neq i}(M)\rangle_{\mathrm{T}}$$
$$= 0. \tag{17}$$

In confirming the above result, $\langle E_i(M)\rangle_{\mathrm{T}} = \langle E_{j\neq i}(M)\rangle_{\mathrm{T}} = \langle E(M)\rangle_{\mathrm{T}}$ is obtained when individ-

ual emission energies are averaged over an ideal black body spectrum. This is shown in Fig. 5, where as we have stated loud and clear repeatedly no correlation among different thermal emissions exists.

For the nonthermal spectrum, the average cross energy term $\langle E_i(M)E_{j\neq i}(M)\rangle$ is nontrivial because of the existence of correlation. It is governed by the probability for two emissions: one at an energy E_i and another at an energy E_j. This probability was derived by us in Ref. [18], where we find that for an extensive list of black holes it satisfies $\Gamma_{\mathrm{NT}}(E_1, E_2) = \Gamma_{\mathrm{NT}}(E_1 + E_2)$. In fact, a recursive use of this relation allows us to show

$$\Gamma_{\mathrm{NT}}(E_1, E_2) = \Gamma_{\mathrm{NT}}(E_1 + E_2)$$
$$= \Gamma_{\mathrm{NT}}(E_1', E_1 + E_2 - E_1'), \tag{18}$$

as long as $E_1 + E_2 - E_1' > 0$, or the probability for emissions $E_1, E_2, E_3, \cdots$ is the same as the probability for the emission of a single radiation with an energy $\sum_j E_j$. This probability distribution is symmetric with respect to any permutation of the individual emission indices. Thus we can work within one sector, and define the normalized probability subjected to the energy conservation constraint $\sum_j E_j \in [0, 8\pi M]$.

The spectrum for multiple emissions thus takes the form,

$$\Gamma_{\mathrm{NT}}(\sum_j E_j) \sim \exp\left[-(M - \sum_j E_j/16\pi)\sum_j E_j\right], \tag{19}$$

which is symmetric with respect to all permutations of indices. $\Gamma_{\mathrm{NT}}(E_1, E_2, E_3 \cdots)$ can be normalized according to $\int_0^{8\pi M} dE_1 \int_0^{8\pi M - E_1} dE_2 \cdots \Gamma_{\mathrm{NT}}(E_1, E_2, E_3 \cdots) = 1$. For the case of two emissions, the explicit form is found to be $\Gamma_{\mathrm{NT}}(E_1, E_2) = \exp\{-[M - (E_1 + E_2)/(16\pi)](E_1 + E_2)\}/8\pi[-1 + e^{-4\pi M^2} + 4\sqrt{\pi}M\mathrm{F}(2\sqrt{\pi}M)]$, which gives

$$\langle E_i(M)E_{j\neq i}(M)\rangle = \frac{8\pi\left[\left(12\pi M^2 - 1\right)e^{-4\pi M^2} + 1 - 4\pi M^2 + 2\sqrt{\pi}M\left(8\pi M^2 - 3\right)\mathrm{F}(2\sqrt{\pi}M)\right]}{3[-1 + e^{-4\pi M^2} + 4\sqrt{\pi}M\mathrm{F}(2\sqrt{\pi}M)]}. \tag{20}$$

Together with the result of Eq. (12) for $\langle E(M)\rangle$, we find

$$\delta E_{\mathrm{NT}}^{2(\mathrm{cov})}(M) = \frac{2\pi}{3\mathrm{F}^2(2\sqrt{\pi}M)\left[e^{-4\pi M^2} - 1 + 4\sqrt{\pi}M\mathrm{F}(2\sqrt{\pi}M)\right]}$$
$$\left[72\sqrt{\pi}M\left(1 - e^{-4\pi M^2}\right)^2\mathrm{F}\left(2\sqrt{\pi}M\right) - 4\left(1 + 60\pi M^2\right)e^{-4\pi M^2}\mathrm{F}^2(2\sqrt{\pi}M)\right.$$
$$\left. + 6\left(1 - e^{-4\pi M^2}\right)^3 + 4\left(1 + 68\pi M^2\right)\mathrm{F}^2(2\sqrt{\pi}M) - 8\sqrt{\pi}M\left(3 + 40\pi M^2\right)\mathrm{F}^3(2\sqrt{\pi}M)\right]. \tag{21}$$

The two limits are given respectively by

$$\delta E_{\mathrm{NT}}^{2(\mathrm{cov})}(M \to 0) \sim -\frac{32\pi^2 M^2}{3} + \frac{96\pi^3 M^4}{5} + \cdots, \tag{22}$$

$$\delta E_{\mathrm{NT}}^{2(\mathrm{cov})}(M \to \infty) \sim -\frac{29}{16\pi M^4}. \tag{23}$$

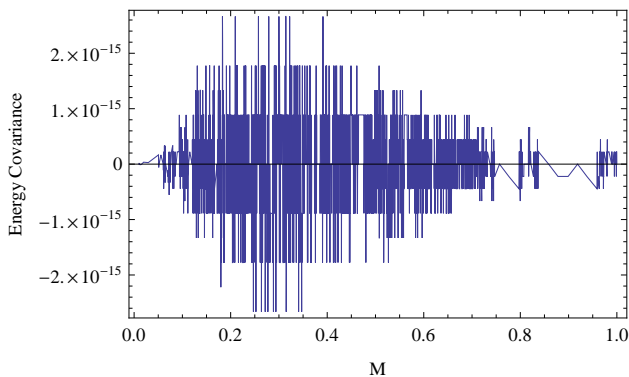Upon checking for the quantitative results of Fig. 6,

FIG. 5: (Color online) The covariances of successive emissions are zero for the thermal spectrum, *i.e.*, there is no correlation among Hawking radiations for the thermal spectrum. The "zero" noise feature is a result of numerical round-off error.
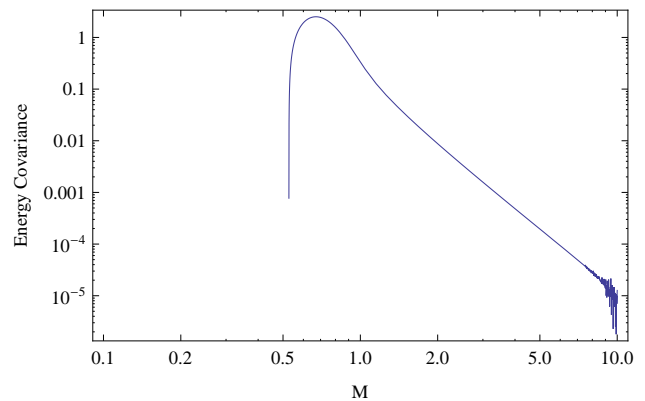

FIG. 6: (Color online) The covariances of successive emissions is non-trivial for the nonthermal spectrum. One can distinguish the nonthermal spectrum from the thermal one by counting the covariances. This provides an avenue towards experimentally testing the paradox.

we find that the covariance approaches its maximum also near the Planck scale. It is interesting that the covariance vanishes at small or large masses. As is discussed in Fig. 3, the number of emissions becomes limited (for instance, 2 emissions) when its mass is small. The covariance thus vanishes at the small mass limit $[M \to 0$ in Eq. (22)]. For large masses, the curve of covariance decreases because the average emission energy becomes small, as illustrated in Fig. 2. The correlation between two emissions E_i and E_j is proportional to the product of E_i and E_j, so it is reasonable that the covariance decreases at large mass limit $[M \to \infty$ in Eq. (23)].

VI. DISCUSSION AND CONCLUSION

In reality, the effective radiation temperature for any astrophysical black hole is much less than the cosmic background temperature. So, the possibility of the Hawking radiation observation focuses on the micro black holes which have been discussed extensively in connection with the experiments of the CERN Large Hadron Collider (LHC) [22–26]. In this regards, the D-dimensional Schwarzschild metric is required and the fundamental Planck scale is reduced depending on the compact space of volume V_{D-4}, e.g. the reduced Planck scale $M_{pl} \sim 1 Tev$ with $D = 10$ and $V_6 \sim fm^6$ [24]. A recent estimate showed that the minimum black hole mass should be in the range of $3.5 - 4.5$ TeV for pp collisions at a center-of-mass energy of 7 TeV at LHC [26]. According to our analysis, the energy covariance (21) for the nonthermal spectrum is larger near the Planck scale, i.e. $\delta E_{\rm NT}^{2({\rm cov})} \sim 1$ for a black hole mass $\sim 0.7 M_{pl}$, which shows that if the radiation of a micro black hole were observed at LHC, its energy covariance will determine whether the emission spectrum is indeed nonthermal. Of course, the energy scale about the production and observation of micro black holes is being debated [27] and so it remains to be seen when and whether the micro black holes could

be observed on Earth, especially in LHC experiment.

On the other hand, observation of Hawking radiation at small man-made black holes such as those implemented or discussed with optical, acoustic, and cold-atomic systems [28–33] are being discussed, and several experiments [29–33] had shown the evidence of the Hawking radiation from the event horizon. For instance, the observation of the analogous Hawking radiation from ultrashort laser pulse filaments was recently reported by creating a traveling refractive index perturbation (RIP) in the fused silica glass [30]. In the experiment, the analogous temperature of Hawking radiation is $T_H \sim 10^{-3} K$ which is equivalent to the radiation of a black hole with mass $M \sim 10^{26} kg$. Such mass is large enough to make the average energy covariance is ignorable, i.e. $\delta E_{\rm NT}^{2({\rm cov})} \sim 10^{-120}$. Actually, the nonthermal probability in real units could be written as $\Gamma_{NT} = \exp\left[-8\pi E\left(M - \frac{E}{2c^2}\right)\frac{G}{\hbar c^3}\right] = \exp\left[-8\pi E M \frac{G}{\hbar c^3}\right] \exp\left[8\pi E^2 \frac{G}{2\hbar c^5}\right] = \Gamma_T \Delta\Gamma$ where $\Delta\Gamma = \exp\left[-8\pi E^2 \frac{G}{2\hbar c^5}\right]$ is the correction factor. If we want to observe the nonthermal spectrum, it is important to make $\Delta\Gamma$ deviate significantly from 1. For the temperature $T_H \sim 10^{-3} K$, the most probable emitted energy is $E \sim 10^{-25} J$, which shows $\Delta\Gamma \sim 1$ and so it is impossible to distinguish the nonthermal and thermal spectra, in this case unless emissions with significantly larger E can be observed. It is estimated that the observable $\Delta\Gamma$ requires the emissions E with ultra-high energy, e.g., $\Delta\Gamma \sim \Gamma_T$ indicates $E \gg 1J$ which cannot happen in the discussed physical system. It is noticed that the analysis here bases on the correspondence between optical black hole and Schwarzschild black hole, so it remains to be seen whether this correspondence is permitted when the backreaction is included in the radiation process and it is still interesting to see whether the correlation between emissions of Hawking radiation could be observed since the unitarity requires the existence of such correlations

[17–19].

In summary, the important differences between the thermal and the nonthermal spectra, such as their respective average emission energies, average numbers of emissions, average emission energy fluctuations, and energy covariances, are calculated, compared, and discussed. Based on our earlier result that so-called lost information of a black hole is encoded into correlations among Hawking radiations, we can verify whether the spectrum of Hawking radiation is nonthermal or not by counting the energy covariances of Hawking radiations, which presents an avenue towards experimentally testing the paradox of black hole information loss.

VII. ACKNOWLEDGEMENT

[1] S. W. Hawking, Nature **248**, 30 (1974).
[2] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[3] R. M. Wald, Phys. Rev. D **13**, 3176 (1976).
[4] D. N. Page, Phys. Rev. D **13**, 198 (1976).
[5] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[6] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[7] L. Susskind, Nature Physics **2**, 665 (2006).
[8] Y. Aharonov, A. Casher, and S. Nussinov, Phys. Lett. B **191**, 51 (1987).
[9] L. M. Krauss and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989).
[10] J. D. Bekenstein, Phys. Rev. Lett. **70**, 3680 (1993).
[11] G. T. Horowitz and J. Maldacena, J. High Energy Phys. **02**, 008 (2004).
[12] S. W. Hawking, Phys. Rev. D **72**, 084013 (2005).
[13] E. D. Belokolos and M. V. Teslyk, Class. Quantum Grav. **26**, 235008 (2009).
[14] P. Kraus and F. Wilczek, Nucl.Phys.B **433**, 403 (1995).
[15] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).
[16] L. Vanzo, G. Acquaviva, and R. Di Criscienzo, Class. Quantum Grav. **28**, 183001 (2011).
[17] B. Zhang, Q.-y. Cai, L. You, and M.-s. Zhan, Phys. Lett. B **675**, 98 (2009).
[18] B. Zhang, Q.-y. Cai, M.-s. Zhan, and L. You, Ann. Phys. **326**, 350 (2011).
[19] W. Israel and Z. Yun, Phys. Rev. D **82**, 124036 (2010).
[20] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[21] M. Abramowitz and I. A. Stegun, (Eds.). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (New York: Dover, 1972), pp. 295 and 319.
[22] R. Emparan, G. T. Horowitz, and R. C. Myers, Phys. Rev. Lett. **85**, 499 (2000).
[23] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001).
[24] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002).
[25] P. Meade and L. Randall, J. High Energy Phys. **05**, 003 (2008).
[26] CMS Collaboration, Phys. Lett. B **697**, 434 (2011).
[27] J. Mureika, P. Nicolini, and E. Spallucci, Phys. Rev. D **85**, 106007 (2012).
[28] W. G. Unruh, Phys. Rev. Lett. **46**, 1351–1353 (1981).
[29] T. G. Philbin, C. Kuklewicz, S. Robertson, S. Hill, F. König, and U. Leonhardt, Science **319**, 1367 (2008).
[30] F. Belgiorno, S. L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi, L. Rizzi, E. Rubino, V. G. Sala, and D. Faccio, Phys. Rev. Lett. **105**, 203901 (2010).
[31] G. Rousseaux, C. Mathis, P. Maïssa, T. G. Philbin, and U. Leonhardt, New J. Phys. 10, 053015 (2008).
[32] S. Weinfurtner, E. W. Tedford, M. C. J. Penrice, W. G. Unruh, and G. A. Lawrence, Phys. Rev. Lett. **106**, 021302 (2011).
[33] O. Lahav, A. Itah, A. Blumkin, C. Gordon, S. Rinott, A. Zayats, and J. Steinhauer, Phys. Rev. Lett. **105**, 240401 (2010).